

Non-Gaussianities of primordial perturbations and tensor sound speed

Toshifumi Noumi^{1,*} and Masahide Yamaguchi^{2,†}

¹*Mathematical Physics Laboratory, RIKEN Nishina Center, Saitama 351-0198, JAPAN*

²*Department of Physics, Tokyo Institute of Technology, Tokyo 152-8551, Japan*

We investigate the relation between the non-Gaussianities of the primordial perturbations and the sound speed of the tensor perturbations, that is, the propagation speed of the gravitational waves. We find that the sound speed of the tensor perturbations is directly related not to the auto-bispectrum of the tensor perturbations but to the cross-bispectrum of the primordial perturbations, especially, the scalar-tensor-tensor bispectrum. This result is in sharp contrast with the case of the scalar (curvature) perturbations, where their reduced sound speed enhances their auto-bispectrum. Our findings indicate that the scalar-tensor-tensor bispectrum can be a powerful tool to probe the sound speed of the tensor perturbations.

Introduction

Inflation is now widely accepted as a paradigm of early Universe to explain the origin of the primordial perturbations as well as to solve the horizon and the flatness problems of the standard big-bang cosmology [1]. The current observational data such as the cosmic microwave background (CMB) anisotropies support almost scale-invariant, adiabatic, and Gaussian primordial curvature fluctuations as predicted by inflation. While the paradigm itself is well established and widely accepted, its detailed dynamics, e.g. the identification of an inflaton, its kinetic and potential structure, and its gravitational coupling, are still unknown.

The non-Gaussianities of primordial curvature perturbations are powerful tools to give such informations. It is well-known that the equilateral type of bispectrum of primordial curvature perturbations is enhanced by the inverse of their sound speed squared [2, 3]. The null observation of the equilateral type by the Planck satellite, characterized as $f_{\text{NL}}^{\text{equil}} = 42 \pm 75$ (68% CL) [4], yields stringent constraints on the sound speed of the curvature perturbations as $c_s \geq 0.02$ (95% CL) [4]. The local type of bispectrum of primordial curvature perturbations also gives useful informations. Maldacena's consistency relation [5, 6] says that the parameter $f_{\text{NL}}^{\text{local}}$ characterizing the local type of bispectrum is as much as $\mathcal{O}(0.01)$ for a single field inflation model.¹ Though the current constraint on this parameter by the Planck satellite is already tight as $f_{\text{NL}}^{\text{local}} = 2.7 \pm 5.8$ (68% CL) [4], the detection of non-Gaussianities with $f_{\text{NL}}^{\text{local}} \sim \mathcal{O}(1)$ by future observations would rule out a single field inflation model.

Inflation generates not only primordial curvature perturbations but also primordial tensor perturbations [8]. Very recently, it was reported that primordial tensor perturbations have been found and the tensor-to-scalar ratio

r is given by $r = 0.20^{+0.07}_{-0.05}$ (68% CL) [9], though it is constrained as $r < 0.11$ (95% CL) in the Planck results [10]. Their amplitude directly determines the energy scale of inflation, so it is estimated as $V^{1/4} \simeq 2.2 \times 10^{16}$ GeV given $r \simeq 0.2$ [9] and $P_\zeta \simeq 2.2 \times 10^{-9}$ [10]. If we go beyond the powerspectrum, it is known that the bispectra of primordial tensor perturbations enable us to probe the gravitational coupling of the inflaton field [11]. Such a non-trivial gravitational coupling easily modifies the sound speed of primordial tensor perturbations, c_γ , and it can significantly deviate from unity [12]. Then, one may wonder if the small sound velocity of primordial tensor perturbations can enhance their non-Gaussianities as in the case of the curvature perturbations. In this *Letter*, we are going to address this issue.

The relation between the sound speed and the non-Gaussianities of primordial curvature perturbations can be clearly understood by use of the effective field theory (EFT) approach to inflation [13]. Inflation can be characterized by the breakdown of time-diffeomorphism invariance due to the time-dependent cosmological background and the general action for inflation can be constructed based on this symmetry breaking structure. The primordial curvature perturbation can be identified with the Goldstone mode π associated with the breaking of time-diffeomorphism invariance. The primordial curvature perturbation can be identified with the Goldstone mode π associated with the breaking of time-diffeomorphism invariance. The symmetry arguments require that modification of the sound speed c_s induces non-negligible cubic interactions of the Goldstone mode π , and hence the sound speed and the bispectrum of the curvature perturbations are directly related.

In this *Letter*, we investigate the relation between the sound speed of tensor perturbations and the bispectrum of primordial perturbations, based on the EFT approach. We first identify what kind of operators can modify the tensor sound speed. Then, we clarify which type of bispectrum arises associated with the modification and can be used as a probe for the tensor sound speed.

¹ This consistency relation is derived under some reasonable assumptions. If we violate some of them, there is a counterexample, which is given in Ref. [7] for example.

The EFT approach

We start from a brief review of the EFT approach [13] and clarify our setup. In the unitary gauge, where the inflaton field does not fluctuate, dynamical degrees of freedom in single-clock inflation are the metric field $g_{\mu\nu}$ only and the action should respect the (time-dependent) spatial diffeomorphism invariance. The action at the lowest order in perturbations can be uniquely determined by the background equations of motion and the residual spatial diffeomorphism invariance as

$$S_0 = M_{\text{Pl}}^2 \int d^4x \sqrt{-g} \left[\frac{1}{2} R + \dot{H} g^{00} - (3H^2 + \dot{H}) \right], \quad (1)$$

where $H(t) = \dot{a}/a$ is the background Hubble parameter with $a(t)$ being the background scale factor

$$ds^2 = -dt^2 + a(t)^2 d\vec{x}^2. \quad (2)$$

This simplest action describes tree-level dynamics of the single-field inflation with a canonical kinetic term in the Einstein gravity. Modifications of inflation models and quantum corrections can be described by including higher order perturbation terms. Ingredients for higher order perturbations are δg^{00} , $\delta K_{\mu\nu}$, $\delta R_{\mu\nu\rho\sigma}$, and their derivatives:

$$\begin{aligned} \delta g^{00} &= g^{00} + 1, \quad \delta K_{\mu\nu} = K_{\mu\nu} - H h_{\mu\nu}, \\ \delta R_{\mu\nu\rho\sigma} &= R_{\mu\nu\rho\sigma} - H^2 (h_{\mu\rho} h_{\nu\sigma} - h_{\mu\sigma} h_{\nu\rho}) \\ &\quad + (H^2 + \dot{H})(h_{\mu\rho} n_\nu n_\sigma + 3 \text{ terms}), \end{aligned} \quad (3)$$

which are covariant under the spatial diffeomorphism and vanish on the FRW background. Here $n_\mu = -\frac{\delta_\mu^0}{\sqrt{-g^{00}}}$ is the unit vector perpendicular to constant- t surfaces, $h_{\mu\nu} = g_{\mu\nu} + n_\mu n_\nu$ is the induced spatial metric, and $K_{\mu\nu} = h_\mu^\sigma \nabla_\sigma n_\nu$ is the extrinsic curvature on the spatial slices. We define the scalar curvature perturbation ζ and the tensor perturbation γ_{ij} as

$$h_{ij} = a^2 e^{2\zeta} (e^\gamma)_{ij} \quad \text{with} \quad \gamma_{ii} = \partial_i \gamma_{ij} = 0. \quad (4)$$

The general action for single-clock inflation can then be expanded in perturbations and derivatives as [13]

$$\begin{aligned} S = S_0 + \int d^4x \sqrt{-g} \left[\frac{M_2(t)^4}{2} (\delta g^{00})^2 - \frac{\bar{M}_1(t)^3}{2} \delta g^{00} \delta K \right. \\ \left. - \frac{\bar{M}_2(t)^2}{2} \delta K^2 - \frac{\bar{M}_3(t)^2}{2} \delta K_\mu^\nu \delta K_\nu^\mu + \dots \right], \end{aligned} \quad (5)$$

where $\delta K = \delta K_\mu^\mu$ and the dots stand for higher derivative terms and higher order perturbations. The above four correction terms are the only operators relevant to the dispersion relations of primordial perturbations in the decoupling limit, with up to two derivatives on metric perturbations, and without higher time derivatives such as $\ddot{\zeta}$. In the following we focus on these operators (see [14] for more general cases).

Tensor sound speed and the powerspectrum

We now investigate the tensor perturbations γ_{ij} based on the EFT framework. Among the operators displayed in (5), only $\delta K_\mu^\nu \delta K_\nu^\mu$ except the Einstein-Hilbert action induces the second order tensor perturbations:

$$\delta K_\mu^\nu \delta K_\nu^\mu \ni -\frac{1}{4} (\partial_t e^{-\gamma} \partial_t e^\gamma)_{ii} = \frac{1}{4} (\dot{\gamma}_{ij})^2 + \mathcal{O}(\gamma^4), \quad (6)$$

which deforms the kinetic term for γ as

$$\int d^4x a^3 \frac{M_{\text{Pl}}^2}{8} c_\gamma^{-2} \left[(\dot{\gamma}_{ij})^2 - c_\gamma^2 \frac{(\partial_k \gamma_{ij})^2}{a^2} \right]. \quad (7)$$

Here the tensor sound speed c_γ is given by

$$c_\gamma^2 = \frac{M_{\text{Pl}}^2}{M_{\text{Pl}}^2 - M_3^2}. \quad (8)$$

To compute the powerspectrum, let us decompose γ_{ij} into the two helicity modes as

$$\gamma_{ij} = \int \frac{d^3k}{(2\pi)^3} \sum_{s=\pm} \epsilon_{ij}^s(\mathbf{k}) \gamma_{\mathbf{k}}^s(t) e^{i\mathbf{k}\cdot\mathbf{x}}, \quad (9)$$

where $s = \pm$ is the helicity index. The polarization tensor $\epsilon_{ij}^s(\mathbf{k})$ is symmetric, traceless, and transverse. Its normalization and reality conditions can be stated as

$$\sum_{i,j} \epsilon_{ij}^s(\mathbf{k}) \epsilon_{ij}^{s'}(-\mathbf{k}) = 2\delta_{ss'}, \quad (\epsilon_{ij}^s(\mathbf{k}))^* = \epsilon_{ij}^s(-\mathbf{k}). \quad (10)$$

These two helicity modes are quantized as

$$\gamma_{\mathbf{k}}^s(t) = b_{s,\mathbf{k}} v_{\mathbf{k}}(t) + b_{s,-\mathbf{k}}^\dagger v_{\mathbf{k}}^*(t) \quad (11)$$

with the standard commutation relation

$$[b_{s,\mathbf{k}}, b_{s',\mathbf{k}'}^\dagger] = \delta_{ss'} (2\pi)^3 \delta(\mathbf{k} - \mathbf{k}'). \quad (12)$$

Here and in what follows, we neglect the time-dependence of the Hubble parameter H and the sound speed c_γ . The mode function $v_{\mathbf{k}}$ for the Bunch-Davies vacuum is then given by

$$v_{\mathbf{k}} = \frac{H}{M_{\text{Pl}}} \frac{c_\gamma}{(c_\gamma k)^{3/2}} (1 + i c_\gamma k \tau) e^{-i c_\gamma k \tau}, \quad (13)$$

where τ is the conformal time $a d\tau = dt$. With this mode function, the tensor two point function is calculated as

$$\begin{aligned} \langle \gamma_{\mathbf{k}}^s \gamma_{\mathbf{k}'}^{s'} \rangle &= \delta_{ss'} (2\pi)^3 \delta^{(3)}(\mathbf{k} - \mathbf{k}') \frac{\pi^2}{2k^3} \mathcal{P}_\gamma(k) \\ \text{with } \mathcal{P}_\gamma(k) &= c_\gamma^{-1} \cdot \frac{2H^2}{\pi^2 M_{\text{Pl}}^2}. \end{aligned} \quad (14)$$

Note that the tensor powerspectrum \mathcal{P}_γ is proportional to c_γ^{-1} , and therefore, the tensor-to-scalar ratio $r = \mathcal{P}_\gamma / \mathcal{P}_\zeta$ has a negative correlation with the tensor sound speed $r \propto c_\gamma^{-1}$.

Tensor bispectrum

Let us then discuss the relation between the tensor sound speed c_γ and the tensor bispectrum. An important point is that no tensor cubic interactions arise from the operator $\delta K_\mu^\nu \delta K_\nu^\mu$ as shown in (6), in contrast to the scalar sound speed case [13]. If we concentrate on the operators displayed in (5), the only source of tensor cubic interactions is the Einstein Hilbert term in S_0 :

$$S_0 \ni M_{\text{Pl}}^2 \int d^4x \frac{a}{4} \left(\gamma_{ik} \gamma_{jl} - \frac{1}{2} \gamma_{ij} \gamma_{kl} \right) \partial_k \partial_l \gamma_{ij}. \quad (15)$$

The deformation of the tensor sound speed can then affect tensor bispectra only through the change in the field normalization and the sound horizon.

For qualitative understanding of these effects, let us first perform an order estimation of the nonlinearity parameter. For this purpose, it is convenient to work in the real coordinate space, rather than in the momentum space. In the real coordinate space, the two-point function is estimated as

$$\langle \gamma \gamma \rangle \sim c_\gamma^2 \frac{H^2}{M_{\text{Pl}}^2}. \quad (16)$$

On the other hand, the three-point function originated from the cubic interaction (15) can be estimated as

$$\langle \gamma \gamma \gamma \rangle \sim \left(\frac{M_{\text{Pl}}^2}{c_\gamma^2 H^2} \right) \cdot \left(c_\gamma^2 \frac{H^2}{M_{\text{Pl}}^2} \right)^3, \quad (17)$$

where the first factor arises from the vertex (15) and we used $\frac{\partial_i}{a} = c_\gamma^{-1} \cdot c_\gamma \frac{\partial_i}{a} \sim c_\gamma^{-1} H$. The second factor is from the three tensor propagators. We can then estimate the nonlinearity parameter (normalized by the tensor powerspectrum) as

$$\tilde{f}_{\text{NL},\gamma} \sim \frac{\langle \gamma \gamma \gamma \rangle}{\langle \gamma \gamma \rangle^2} \sim 1, \quad (18)$$

which is of the order one and independent of the tensor sound speed c_γ .

For more details, we present the result of the momentum space analysis briefly. Taking into account the modification of the field normalization and the sound horizon, we can easily factorize the c_γ -dependence of the bispectrum as

$$\langle \gamma_{\mathbf{k}_1}^{s_1} \gamma_{\mathbf{k}_2}^{s_2} \gamma_{\mathbf{k}_3}^{s_3} \rangle = c_\gamma^{-2} \cdot \langle \gamma_{\mathbf{k}_1}^{s_1} \gamma_{\mathbf{k}_2}^{s_2} \gamma_{\mathbf{k}_3}^{s_3} \rangle \Big|_{S_0}, \quad (19)$$

where $\langle \gamma_{\mathbf{k}_1}^{s_1} \gamma_{\mathbf{k}_2}^{s_2} \gamma_{\mathbf{k}_3}^{s_3} \rangle \Big|_{S_0}$ represents the three-point function taking into account only S_0 (that is, $c_\gamma = 1$ case) [5]. In terms of the shape function $\tilde{S}_{s_1, s_2, s_3}$ normalized by the tensor powerspectrum,

$$\begin{aligned} & \langle \gamma_{\mathbf{k}_1}^{s_1} \gamma_{\mathbf{k}_2}^{s_2} \gamma_{\mathbf{k}_3}^{s_3} \rangle \\ &= (2\pi)^3 \delta^{(3)} \left(\sum_i \mathbf{k}_i \right) \frac{(2\pi)^4 \mathcal{P}_\gamma^2}{k_1^2 k_2^2 k_3^2} \tilde{S}_{s_1, s_2, s_3}(k_1, k_2, k_3), \quad (20) \end{aligned}$$

the tensor bispectrum can then be expressed as²

$$\begin{aligned} \tilde{S}_{s_1, s_2, s_3}(k_1, k_2, k_3) &= \frac{\sqrt{2}}{8192} \frac{F(s_1 k_1, s_2 k_2, s_3 k_3)}{k_1^3 k_2^3 k_3^3} \\ &\times \left[k_t - \frac{\sum_{i>j} k_i k_j}{k_t} - \frac{k_1 k_2 k_3}{k_t^2} \right], \quad (21) \end{aligned}$$

where $k_t = k_1 + k_2 + k_3$ and $F(x, y, z)$ is given by

$$\begin{aligned} F(x, y, z) &= (x + y + z)^5 (x + y - z)(y + z - x)(z + x - y). \quad (22) \end{aligned}$$

The point is that the c_γ -dependence of the three-point function can be absorbed into the prefactor \mathcal{P}_γ^2 in (20) and the shape function is c_γ -independent. The nonlinearity parameter defined by

$$\tilde{f}_{\text{NL},\gamma}^{s_1 s_2 s_3} = \frac{10}{9} \tilde{S}_{s_1, s_2, s_3}(k, k, k) \quad (23)$$

can be also calculated as [15]

$$\tilde{f}_{\text{NL},\gamma}^{\pm\pm\pm} = \frac{255\sqrt{2}}{4096}, \quad \tilde{f}_{\text{NL},\gamma}^{\pm\pm\mp} = \frac{85\sqrt{2}}{110592}. \quad (24)$$

As we already discussed in the qualitative estimation, $\tilde{f}_{\text{NL},\gamma}$ does not depend on the tensor sound speed c_γ and is of the order one. Also note that the nonlinearity parameter $f_{\text{NL},\gamma}$ normalized by the scalar powerspectrum is given by

$$f_{\text{NL},\gamma} = \frac{\mathcal{P}_\gamma^2}{\mathcal{P}_\zeta^2} \tilde{f}_{\text{NL},\gamma} = r^2 \tilde{f}_{\text{NL},\gamma} \sim r^2. \quad (25)$$

To summarize, the relations (24) and (25) for the nonlinearity parameters do not depend on the sound speed c_γ explicitly. The shape of bispectra is also independent of the tensor sound speed essentially because the operator $\delta K_\mu^\nu \delta K_\nu^\mu$ does not induce tensor cubic interactions. In this sense, we cannot identify the tensor sound speed only from the tensor bispectrum. In particular, a large $\tilde{f}_{\text{NL},\gamma}$ cannot be obtained unless other operators of higher dimension or higher order perturbations are relevant.

Importance of cross correlations

As we have discussed, it is not possible to determine the tensor sound speed only from the tensor powerspectrum and bispectrum. We now show that cross correlations can be a useful probe for the tensor sound speed. For this purpose, let us perform the Stückelberg method

² Here and in what follows, we drop a phase factor associated with the spin-2 structure of the polarization tensor for simplicity. See [14] for details.

and introduce the Goldstone boson π associated with the breaking of time diffeomorphism invariance. By a field-dependent coordinate transformation

$$(t, x^i) \rightarrow (\tilde{t}, \tilde{x}^i) \quad \text{with} \quad \tilde{t} + \tilde{\pi}(\tilde{t}, \tilde{x}) = t, \quad \tilde{x}^i = x^i, \quad (26)$$

the minimal action S_0 is transformed as

$$S_0 = M_{\text{Pl}}^2 \int d^4x a^3 \left[-\dot{H} \left(\dot{\pi}^2 - \frac{(\partial_i \pi)^2}{a^2} + \frac{\gamma_{ij} \partial_i \pi \partial_j \pi}{a^2} \right) + \frac{1}{8} \left(\dot{\gamma}_{ij}^2 - \frac{(\partial_k \gamma_{ij})^2}{a^2} \right) + \frac{1}{8} (2\gamma_{ik} \gamma_{jl} - \gamma_{ij} \gamma_{kl}) \frac{\partial_k \partial_l \gamma_{ij}}{a^2} \right]. \quad (27)$$

Here we dropped the fluctuations of the lapse and shift, i.e. took the decoupling limit, because their contributions to bispectra are higher order in the slow-roll parameter $\epsilon = -\dot{H}/H^2$ or the couplings M_i 's and \bar{M}_i 's. Also note that the relation between the Goldstone boson π and the scalar perturbation ζ is given by $\zeta = -H\pi$ at the linear order. Similarly, the $\delta K_\mu^\nu \delta K_\nu^\mu$ term is transformed in the decoupling limit as

$$\begin{aligned} & \int d^4x \sqrt{-g} \frac{-\bar{M}_3^2}{2} \delta K_\mu^\nu \delta K_\nu^\mu \\ & \rightarrow \int d^4x a^3 \frac{-\bar{M}_3^2}{2} \left[\frac{(\partial^2 \pi)^2}{a^4} + \frac{1}{4} \dot{\gamma}_{ij}^2 - \frac{1}{2} \dot{\gamma}_{ij} \frac{\partial_k \gamma_{ij} \partial_k \pi}{a^2} \right. \\ & - 2\gamma_{ij} \frac{\partial_i \partial_j \pi \partial_k^2 \pi}{a^4} - \frac{1}{2} \left(\dot{\gamma}_{ij} + H \dot{\gamma}_{ij} - \frac{\partial_k^2 \gamma_{ij}}{a^2} \right) \frac{\partial_i \pi \partial_j \pi}{a^2} \\ & \left. + \dot{\pi} \frac{4\partial^2 \partial_i \pi \partial_i \pi - 2(\partial_i \partial_j \pi)^2 + 4(\partial^2 \pi)^2}{a^4} + \dots \right], \quad (28) \end{aligned}$$

where the dots stand for higher order terms in perturbations and terms proportional to $\dot{\bar{M}}_3$. We notice that (28) contains scalar-tensor-tensor type cubic interactions as well as scalar-scalar-tensor and scalar-scalar-scalar type interactions. In Table I, we summarize what types of interactions arise in the decoupling limit from the operators in (5). There, we find that the scalar-tensor-tensor interaction, the $\gamma^2 \pi$ -type interaction, arises only from the operator $\delta K_\mu^\nu \delta K_\nu^\mu$, while $\gamma \pi^2$ - and π^3 -type interactions arise also from other operators. In this sense, we could say that the scalar-tensor-tensor bispectrum is sensitive to the tensor sound speed c_γ and is enhanced by the deformation of c_γ .

Evaluation of scalar-tensor-tensor bispectrum

We then take a closer look at the scalar-tensor-tensor cross correlations by a concrete in-in formalism computation. By using the relation $\zeta = -H\pi$, the scalar-tensor-tensor correlation can be expressed in terms of the Goldstone boson π as

$$\langle \zeta_{\mathbf{k}_1} \gamma_{\mathbf{k}_2}^{s_2} \gamma_{\mathbf{k}_3}^{s_3} \rangle = -H \langle \pi_{\mathbf{k}_1} \gamma_{\mathbf{k}_2}^{s_2} \gamma_{\mathbf{k}_3}^{s_3} \rangle,$$

operator	$\dot{\pi}^2$	$\frac{(\partial_i \pi)^2}{a^2}$	$\frac{(\partial_i^2 \pi)^2}{a^4}$	$(\dot{\gamma}_{ij})^2$	$\frac{(\partial_k \gamma_{ij})^2}{a^2}$	γ^3	$\gamma^2 \pi$	$\gamma \pi^2$	π^3
S_0	✓	✓		✓	✓	✓		✓	
$(\delta g^{00})^2$	✓								✓
$\delta g^{00} \delta K$		✓						✓	✓
$(\delta K)^2$			✓					✓	✓
$\delta K_\mu^\nu \delta K_\nu^\mu$			✓	✓			✓	✓	✓

TABLE I: Operators relevant to dispersion relations of primordial perturbations and the induced cubic interactions in the decoupling limit.

whose source is the following interaction in (28):

$$\int d^4x a^3 \frac{\bar{M}_3^2}{4} \dot{\gamma}_{ij} \frac{\partial_k \gamma_{ij} \partial_k \pi}{a^2}. \quad (29)$$

As given in Table I, the kinetic term of π can be modified by various correction terms. However, for simplicity, let us take the free theory action for π as

$$-M_{\text{Pl}}^2 \int d^4x a^3 \dot{H} \left(\dot{\pi}^2 - \frac{(\partial_i \pi)^2}{a^2} \right). \quad (30)$$

The Goldstone boson π is then quantized as

$$\pi = \int \frac{d^3k}{(2\pi)^3} \left[a_{\mathbf{k}} u_k(t) + a_{-\mathbf{k}}^\dagger u_k^*(t) \right] e^{i\mathbf{k} \cdot \mathbf{x}}, \quad (31)$$

with the standard commutation relation

$$[a_{\mathbf{k}}, a_{\mathbf{k}'}^\dagger] = (2\pi)^3 \delta(\mathbf{k} - \mathbf{k}') \quad (32)$$

and the Bunch-Davies mode function

$$u_k = \frac{1}{2M_{\text{Pl}} \epsilon^{1/2} k^{3/2}} (1 + ik\tau) e^{-ik\tau}. \quad (33)$$

Introducing the shape function $S_{s_2, s_3}(k_1, k_2, k_3)$ for the scalar-tensor-tensor bispectrum (normalized by the scalar powerspectrum) as

$$\begin{aligned} & \langle \zeta_{\mathbf{k}_1} \gamma_{\mathbf{k}_2}^{s_2} \gamma_{\mathbf{k}_3}^{s_3} \rangle \\ & = (2\pi)^3 \delta^3 \left(\sum_i \mathbf{k}_i \right) \frac{(2\pi)^4 \mathcal{P}_\zeta^2}{k_1^2 k_2^2 k_3^2} S_{s_2, s_3}(k_1, k_2, k_3), \quad (34) \end{aligned}$$

we can easily calculate it by the in-in formalism as

$$S_{s_2, s_3}(k_1, k_2, k_3) = \epsilon_{ij}^{s_2}(\mathbf{k}_2) \epsilon_{ij}^{s_3}(\mathbf{k}_3) \tilde{S}(k_1, k_2, k_3). \quad (35)$$

Here \tilde{S} is a helicity-independent part given by

$$\begin{aligned} & \tilde{S}(k_1, k_2, k_3) \\ & = \epsilon (c_\gamma^{-2} - 1) \frac{(\mathbf{k}_1 \cdot \mathbf{k}_2) k_3^2}{2k_1 k_2 k_3} \left(\frac{1}{K} + \frac{k_1 + c_\gamma k_2}{K^2} + \frac{2k_1 k_2}{K^3} \right) \\ & \quad + (\mathbf{k}_2 \leftrightarrow \mathbf{k}_3), \quad (36) \end{aligned}$$

where $K = k_1 + c_\gamma k_2 + c_\gamma k_3$ and we used $\bar{M}_3^2 = (1 - c_\gamma^{-2})M_{\text{Pl}}^2$. An explicit form of the helicity part is

$$\epsilon_{ij}^{s_2}(\mathbf{k}_2)\epsilon_{ij}^{s_3}(\mathbf{k}_3) = \begin{cases} \frac{1}{2}(1 - \cos\theta)^2 & \text{for } s_2 = s_3, \\ \frac{1}{2}(1 + \cos\theta)^2 & \text{for } s_2 \neq s_3, \end{cases} \quad (37)$$

where θ is the angle between the momenta \mathbf{k}_2 and \mathbf{k}_3 . For $s_2 = s_3$, the helicity part (37) takes its maximum value 2 when \mathbf{k}_2 and \mathbf{k}_3 are antiparallel and vanishes when they are parallel. For $s_2 = -s_3$, vice versa. The total shape function S_{s_2, s_3} can then be classified into $S_{\pm\pm}$ and $S_{\pm\mp}$. As depicted in Fig. 1, the peak of $S_{\pm\pm}$ is around $(k_1, k_2, k_3) \simeq (c_\gamma k, k, k)$, where the three modes have the same sound horizon size. The c_γ -dependence at this point is given by

$$S_{\pm, \pm}(c_\gamma k, k, k) = -\frac{(1 - c_\gamma^2)(4 - c_\gamma^2)^2(2 + 15c_\gamma)}{432c_\gamma^3}\epsilon. \quad (38)$$

On the other hand, as shown in Fig. 2, $S_{\pm, \mp}$ has a peak at $(k_1, k_2, k_3) = (2k, k, k)$, where \mathbf{k}_2 and \mathbf{k}_3 are parallel and the helicity part (37) is maximized. The c_γ -dependence of the peak size is given by

$$S_{\pm, \mp}(2k, k, k) = -\frac{(1 - c_\gamma)(6 + 7c_\gamma + 3c_\gamma^2)}{2c_\gamma^2(1 + c_\gamma)^2}\epsilon. \quad (39)$$

It should be noted that, when the tensor sound speed is unity, the scalar-tensor-tensor bispectrum vanishes in the decoupling limit. Then, the leading contribution comes from fluctuations of the lapse and shift, and becomes $\mathcal{O}(\epsilon^2)$, which is higher order in the slow-roll parameter ϵ compared to the case with $c_\gamma \neq 1$. The scalar-tensor-tensor bispectrum is then enhanced as

$$\frac{\langle \zeta \gamma \gamma \rangle_{c_\gamma \neq 1}}{\langle \zeta \gamma \gamma \rangle_{c_\gamma = 1}} \sim \mathcal{O}\left(\frac{c_\gamma^{-2} - 1}{\epsilon}\right), \quad (40)$$

when the tensor sound speed is modified. Because of the factor $1/\epsilon$, a significant enhancement can occur even if the deformation of the tensor sound speed is not so large, say $c_\gamma \sim 0.8$. Therefore, the enhancement of the scalar-tensor-tensor bispectrum can be a powerful probe for the modified tensor sound speed.

On scalar bispectrum

Finally, we make a brief comment on scalar bispectra induced by the modified tensor sound speed. As in (28), the $\delta K_\mu^\nu \delta K_\nu^\mu$ operator induces cubic interactions of π , which can be a source of large scalar non-Gaussianities. Indeed, if $\delta K_\mu^\nu \delta K_\nu^\mu$ is the only relevant operator and other operators do not come into the game, the scalar nonlinearity parameter f_{NL} can be estimated as

$$f_{\text{NL}} \sim \mathcal{O}((c_\gamma^{-2} - 1)/\epsilon). \quad (41)$$

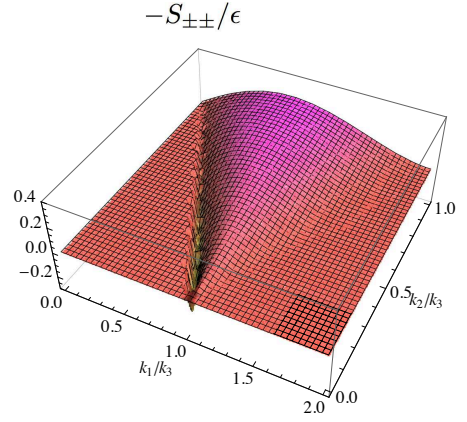


FIG. 1: Shape function $S_{\pm\pm}(k_1, k_2, k_3)$ for $c_\gamma = 0.8$

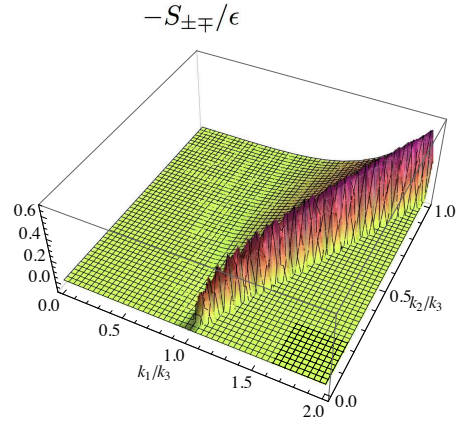


FIG. 2: Shape function $S_{\pm\mp}(k_1, k_2, k_3)$ for $c_\gamma = 0.8$

Since it is enhanced by the inverse of the slow-roll parameter, one may think that the current null observation of scalar non-Gaussianities can constrain the tensor sound speed as $c_\gamma^{-2} - 1 \lesssim \epsilon$. However, as shown in Table I, various operators can induce π^3 -type interactions and the scalar non-Gaussianities can be easily reduced in the presence of other operators. For example, when the δK^2 operator is relevant and $\bar{M}_2^2 = -\bar{M}_3^2$,

$$S = S_0 - \frac{\bar{M}_3^2}{2} \int d^4x \sqrt{-g} (\delta K_\mu^\nu \delta K_\nu^\mu - \delta K^2), \quad (42)$$

the cubic interactions of π exactly cancel out in the decoupling limit.³ In fact, the generalized Galileon [16, 17] accommodates this type of combination in the action [18].

³ If we go beyond the decoupling limit, the leading contributions to the scalar bispectrum may arise from terms with fluctuations of the lapse and shift, which are higher order in $c_\gamma^{-2} - 1$ or ϵ . While such contributions are negligible as long as $c_\gamma^{-2} - 1$ is small, they become relevant when $c_\gamma \lesssim \frac{1}{\sqrt{2}}$ and more careful discussions are required in such a parameter region. See [14] for details.

Thus, additional symmetries or tunings may decrease the scalar non-Gaussianities. In this way, scalar bispectra depend on various operators and the information of the tensor sound speed is obscured. In contrast, the scalar-tensor-tensor bispectra are more sensitive to the tensor sound speed and can be a powerful tool to measure it.

Conclusion

In this *Letter*, we investigated the relation between non-Gaussianities of primordial perturbations and the sound speed of tensor perturbations, based on the EFT approach. We found that the modification of the tensor sound speed induces a significant enhancement of the scalar-tensor-tensor cross bispectrum, rather than the tensor auto-bispectrum. This situation is in sharp contrast with the case of the curvature perturbations, in which their auto-bispectra are significantly enhanced by their reduced sound velocity. When the sound speed of tensor perturbations is reduced, the scalar-tensor-tensor bispectrum is enhanced by a factor of $(c_\gamma^{-2} - 1)/\epsilon$ compared to the case of $c_\gamma = 1$ and such an enhancement makes it easy to detect the CMB bispectra of two B-modes and one temperature (or one E-mode) anisotropies especially. Thus, the scalar-tensor-tensor bispectrum and its relevant bispectra of the CMB can be powerful probes for the reduced tensor sound speed and the gravitational structure for inflation.

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[†] Email: gucci”at”phys.titech.ac.jp

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* Email: toshifumi.noumi”at”riken.jp